Reducing perceived inequalities tends to increase the average opinion about each other

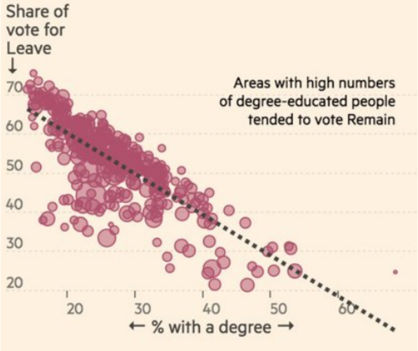
G. Deffuant, T. Roubin

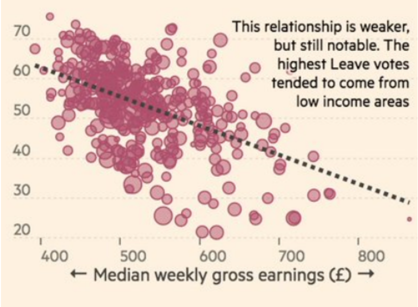
Inrae - Laboratoire d'Ingéniérie des Systèmes Complexes

April 2021

Inequalities and polarization: vote for Brexit

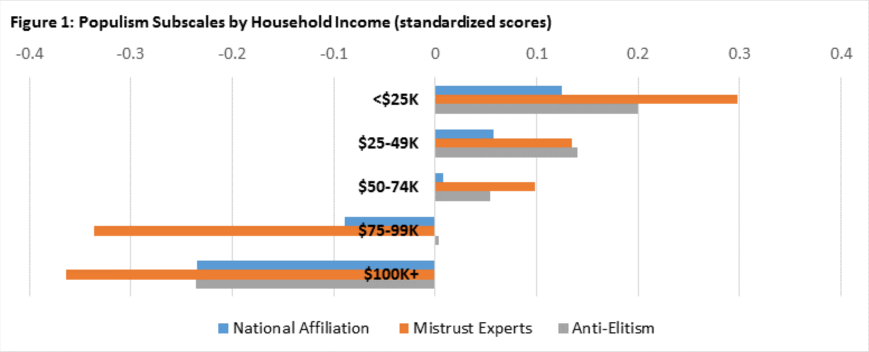
- Data about 382 voting areas:





Inequalities and polarization: Indicators of populism in USA

- National affiliation, mistrust experts, anti-elitism in 2016:

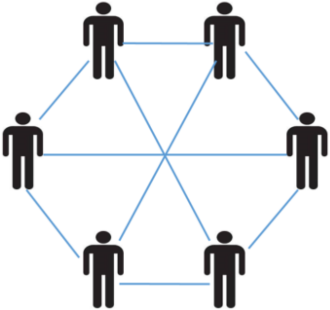


Modelling connection between perceived inequalities and antagonism in society?

- Hypothesis:
  - perceived inequalities generates antagonism in society
  - Antagonism leads to polarization of opinions on various issues
- Our approach: designing a theoretical opinion dynamics model exploring this hypothesis

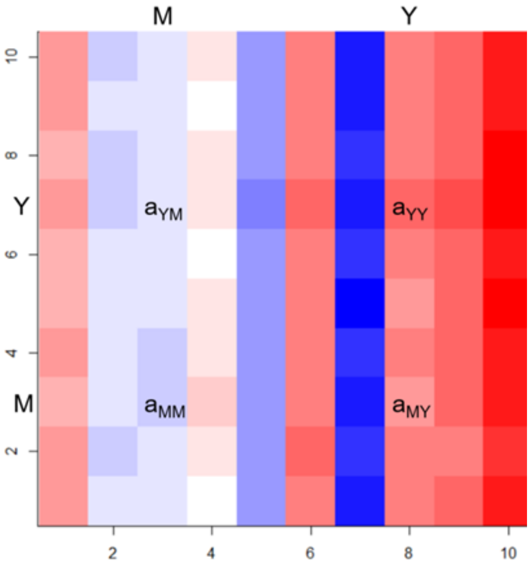
Simple model focusing on effect of interactions

- All agents are initially identical and fully connected
- Only the randomness of the interactions generates diversity
- Objective: better understanding the effect of interactions on perceived inequalities



Model state

- N_a agents
- Every agent M has an opinion a_{MY} about agent Y (including a self-opinion a_{MM}).



Random pair interaction between M and Y

- At each time step, a pair (M, Y) of agents is randomly chosen, then:
  - Agent Y influences agent M
    - Y expresses her opinion about M and Y and possibly k agents H chosen at random (if gossips)
    - This modifies $a_{MM}(t)$, $a_{MY}(t)$ and $a_{MH}(t)$:
  - Agent M influences agent Y
    - M expresses her opinion about M and Y and possibly k agents H' chosen at random (if gossips)
    - This modifies $a_{YM}(t)$, $a_{YY}(t)$ and $a_{YH'}(t)$:

Rule for the modification of opinions

If Y talks about J to M :

- modification of $a_{MJ}(t)$ given by:

$$\Delta a_{MJ}(t) = p_{MY}(t)(a_{YJ}(t) + \theta(t) - a_{MJ}(t))$$

- with:
    - $\theta(t)$ number uniformly drawn between $-\delta$ and δ
    - $p_{MY}(t)$ logistic function:

$$p_{MY}(t) = \frac{1}{1 + \exp\left(\frac{a_{MM}(t) - a_{MY}(t)}{\sigma}\right)}$$

Y expresses a_{YM}, a_{YY} and a_{YH}

	M	Y	H	H'
M	a_{MM} $+$ Δa_{MM}	a_{MY} $+$ Δa_{MY}	a_{MH} $+$ Δa_{MH}	$a_{MH'}$
Y	a_{YM}	a_{YY}	a_{YH}	$a_{YH'}$

M expresses a_{MY}, a_{MM} and $a_{MH'}$

	M	Y	H	H'
M	a_{MM} $+$ Δa_{MM}	a_{MY} $+$ Δa_{MY}	a_{MH} $+$ Δa_{MH}	$a_{MH'}$
Y	a_{YM} $+$ Δa_{YM}	a_{YY} $+$ Δa_{YY}	a_{YH}	$a_{YH'}$ $+$ $\Delta a_{MH'}$

Parameters of the dynamics

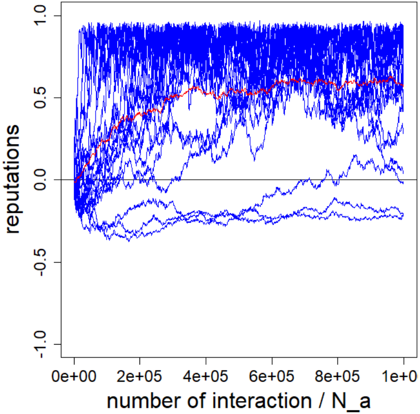
- σ defines the shape of the propagation function p_{MY};
- δ represents the amplitude of the uniformly distributed errors that perturb the evaluation of others' expressed opinions;
- k number of agents subject of gossip at each pair interaction.

- In the following simulations : $\sigma = 0.3$ and $\delta = 0.1$

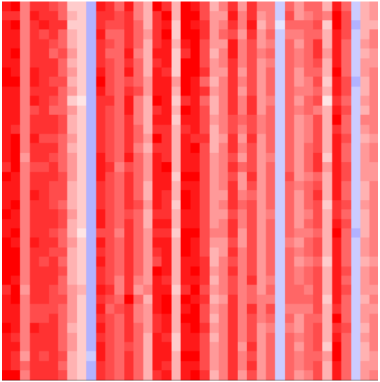
Main patterns of the model

- Without gossip
- With gossip
- When artificially limiting inequalities

Typical pattern without gossip (k=0)

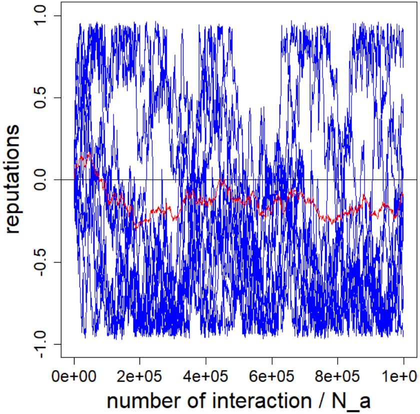


Reputations and average opinion (in red) over time.

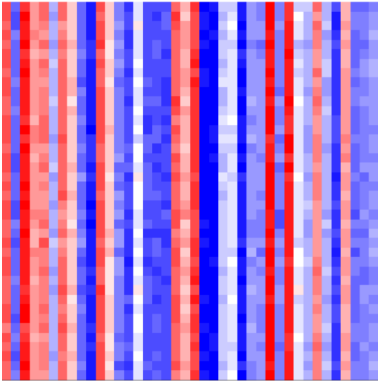


State after 1M $\times N_a$ pair interactions

Typical pattern with gossip (k=5)

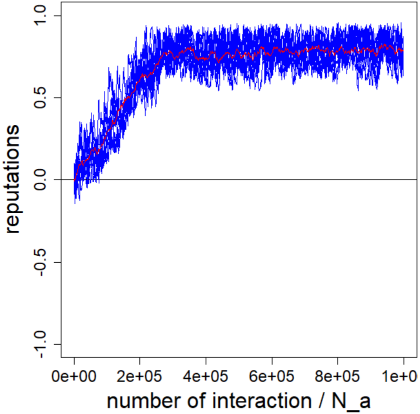


Reputations and average opinion (in red) over time.

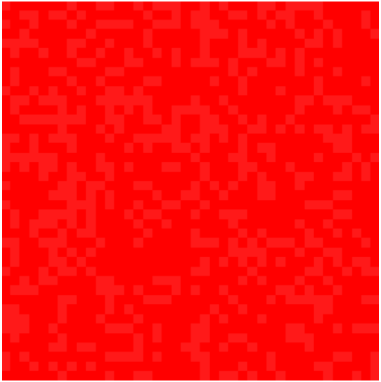


State after 1M $\times N_a$ pair interactions

With gossip (k=5), and inequalities of reputations limited artificially



Reputations and average opinion (in red) over time.



State after 1M $\times N_a$ pair interactions

Theoretical approach: average model over noise and interactions

- Only two agents: positive bias on self-opinion and negative bias on the opinion about others
- Evolution of average reputation without gossip
- Evolution of average reputation with gossip

Positive bias on self-opinions

- Two agents : M and Y;
- The opinions about Y are fixed: $a_{MY}(t) = b$, $a_{YY}(t) = b$
- The opinion of Y about M is fixed $a_{YM}(t) = a$
- Only the self-opinion of M changes because of the noise in messages on the opinion of Y about M, with :
  - $a_{MM}(0) = a$
  - $m(t) = a_{MM}(t) - a$

At the first step, the average of M's self-opinion is zero

- Let :
  - $h = H(a,b) = p_{MY}(0)$: the starting influence of Y on M;
  - $\theta(t)$: noise at time t, drawn from the uniform distribution between $-\delta$ and δ
- We have:

$$m(1) = m(0) + h(\theta(1) - m(0))$$
$$m(1) = h\theta(1)$$

- Therefore, $\overline{m}(1)$ the average of $m(1)$, is:

$$\overline{m}(1) = h\frac{1}{2\delta}\int_{-\delta}^{+\delta}\theta d\theta$$
$$= 0$$

At the second step, the average self-opinion is slightly positive (positive bias)

- Applying the rule of interaction :

$$m(2) = m(1) + H(a + m(1), b)(\theta(2) - m(1))$$

- Developing the influence function at the first order:

$$H(a + m(1), b) \approx h + h' m(1)$$

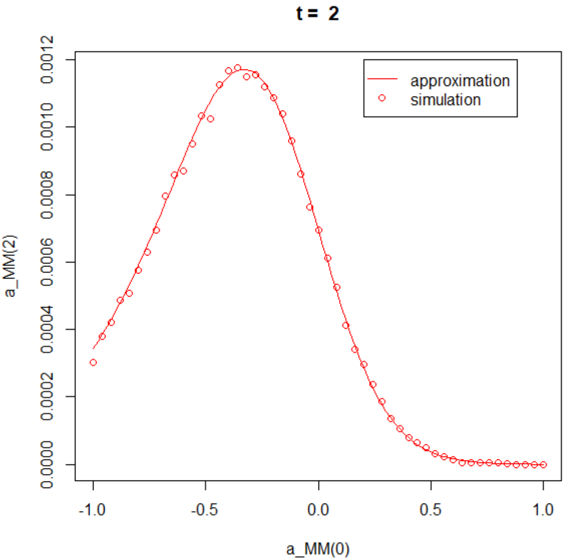
- We get, with $m(1) = h\theta(1)$:

$$\begin{aligned} m(2) &\approx h\theta(1) + (h + h' h\theta(1))(\theta(2) - h\theta(1)) \\ &\approx (1 - h)h\theta(1) + h\theta(2) - h' h^2\theta^2(1) + h' h\theta(1)\theta(2) \end{aligned}$$

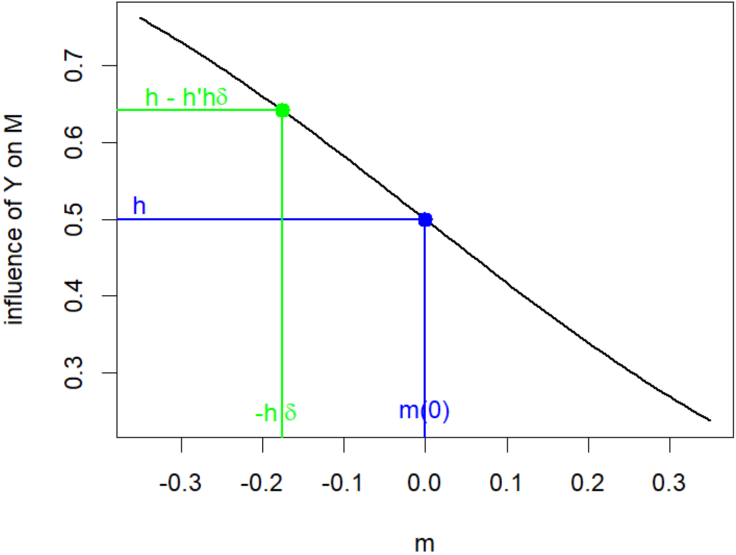
- Averaging over all possible values of $\theta(1)$ and $\theta(2)$

$$\overline{m}(2) = -h' h^2 \frac{1}{2\delta} \int_{-\delta}^{+\delta} \theta^2 d\theta = \frac{-h' h^2 \delta^2}{3} > 0$$

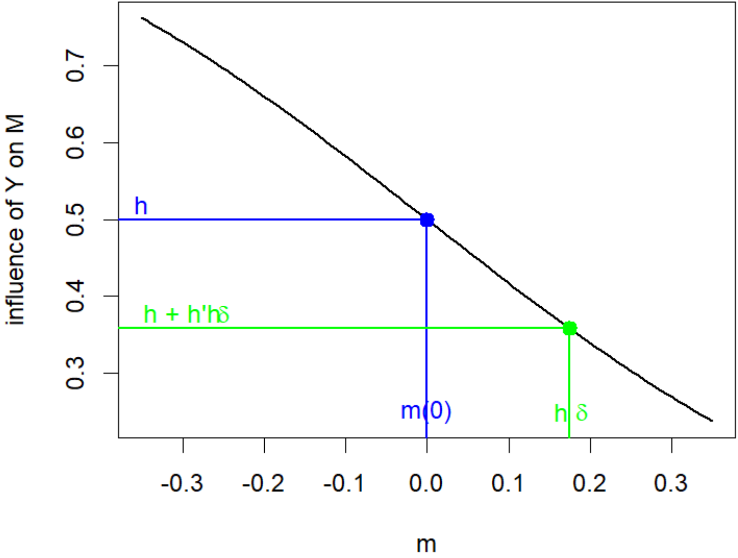
Simulation vs Approximation of $\overline{m}(2)$ for $a_{YY} = a_{MY} = b = 0$



Positive bias: when $m(1)$ is down, $m(2)$ is more easily up



Positive bias: when $m(1)$ is up, $m(2)$ is less easily down



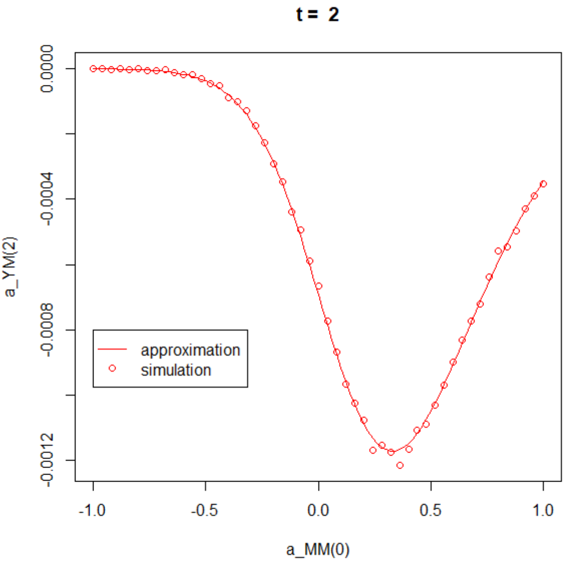
Negative bias on the opinion about others

- M keeps the constant self-opinion $m(0) = a$
- the opinions about Y are fixed:
  - $a_{YY}(t) = b$
  - $a_{MY}(t) = b$
- The-opinion of Y about M changes only because of the noise on M's messages about its self opinion. We set $a_{YM}(0) = y(0) = a$

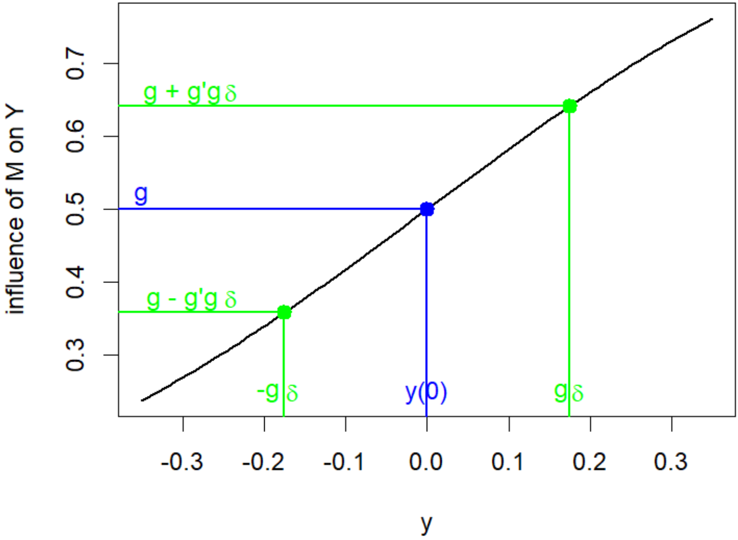
We can calculate $\overline{y}(2)$ the same way, we replace h by $1 - h$ and h' by $-h'$. We obtain :

$$\overline{y}(2) = \frac{h'(1-h)^2\delta^2}{3} < 0 \qquad (1)$$

Simulation vs Approximation of $\overline{y}(2)$ for $a_{YY} = a_{MY} = 0$



Negative bias on the opinion about others: illustration



Approximation of the evolution of average opinions about an agent M

- one agent M and $N_a - 1$ agents Y_i with $i \in \{2, .., N_a\}$,
- only the opinions about M ($a_{MM}(t)$ and $a_{Y_iM}(t)$) change
- the other opinions are assumed fixed ($a_{MY_i}(t) = b_i$ and $a_{Y_jY_i}(t) = b_i$)
- we assume $a_{MM}(0) = a$, $a_{Y_iM}(0) = a$ for $i \in \{2, \ldots, N_a\}$
- we developed a model approximating:
  - $\overline{m}(t)$ the average of $a_{MM}(t) - a_{MM}(0)$ over noise and interactions.
  - $\overline{y_i}(t)$ the average of $a_{Y_iM}(t) - a_{Y_iM}(0)$ over noise and interactions.
- we define the reputation of agent M is:

$$\overline{r_M}(t) = \frac{1}{N_a}\left(\overline{m}(t) + \sum_{i=2}^{N_a} \overline{y}_i(t)\right)$$

Approximation of the evolution of average reputation of an agent without gossip

- From the evolution of $\overline{m}(t)$ and $\overline{y_i}(t)$, we get the evolution of the average reputation:

$$\overline{r_M}(t+1) - \overline{r_M}(t) = \Delta r_M(t) = \Delta_1 r_M(t) + \Delta_2 r_M(t)$$

- with $(h_{(1,i)} = H(a, b_i))$:

$$\Delta_1 r_M(t) = \sum_{i=2}^{N_a} \frac{(1 - 2h_{(1,i)})\left(\overline{m}(t) - \overline{y}_i(t)\right)}{N_a(N_a - 1)}$$

$$\Delta_2 r_M(t) = -\sum_{i=2}^{N_a} \frac{h'_{(1,i)}\left(\overline{m^2}(t) - \overline{y_i^2}(t)\right)}{N_a(N_a - 1)}$$

Evolution of average reputation without gossip at $t = 2$

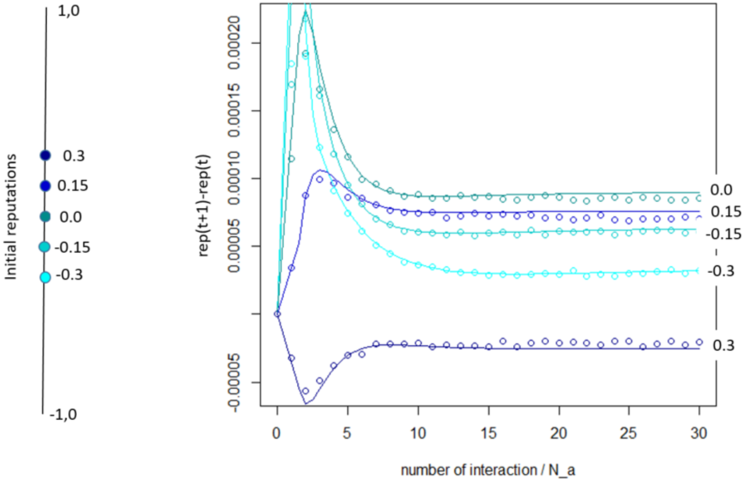
- At $t = 2$, we have:

$$\overline{r_M}(2) = \Delta_2 r_M(2) = -\sum_{i=2}^{N_a} \frac{h'_i\left(\overline{m^2}(1) - \overline{y_i^2}(1)\right)}{N_a(N_a-1)}$$

$$= \frac{1}{N_a(N_a-1)} \sum_{i=2}^{N_a} \left(\left(\frac{-1}{N_a - 1} \sum_{j=2}^{N_a} h'_i h_j^2 \frac{\delta^2}{3} \right) + h'_i (1 - h_i)^2 \frac{\delta^2}{3} \right)$$

- We recognise the negative biases and an average of positive biases for two agents at $t = 2$.

$\Delta r_M(t)$ rapidly stabilises



Lines: analytical approximation, dots: average over 100 million simulations

Approximation of the evolution of average reputation of an agent with gossip

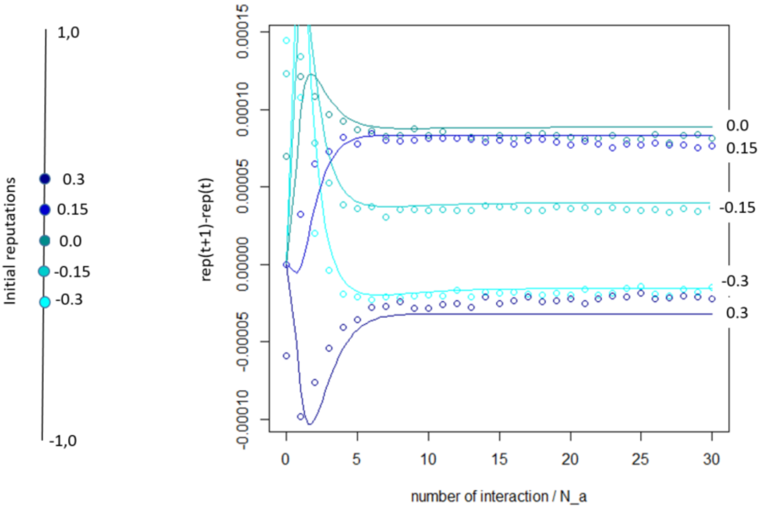
- From the evolution of $\overline{m}(t)$ and $\overline{y_i}(t)$, we get the evolution of the average reputation:

$$\overline{r_M}(t+1) - \overline{r_M}(t) = \Delta r_M(t) = \Delta_1 r_M(t) + \Delta_2 r_M(t) + \Delta_{1g} r_M(t)$$

- with ($h_{(i,i)} = H(b_i, b_j)$):

$$\Delta_{1g} r_M(t) = \sum_{i=2}^{N_a-1} \sum_{j=i+1}^{N_a} \frac{\left(1 - 2h_{(i,j)}\right)\left(\overline{y_i}(t) - \overline{y}_j(t)\right)}{N_a(N_a-1)(N_a-2)}$$

$\Delta r_M(t)$ rapidly stabilises (like previously without gossip)

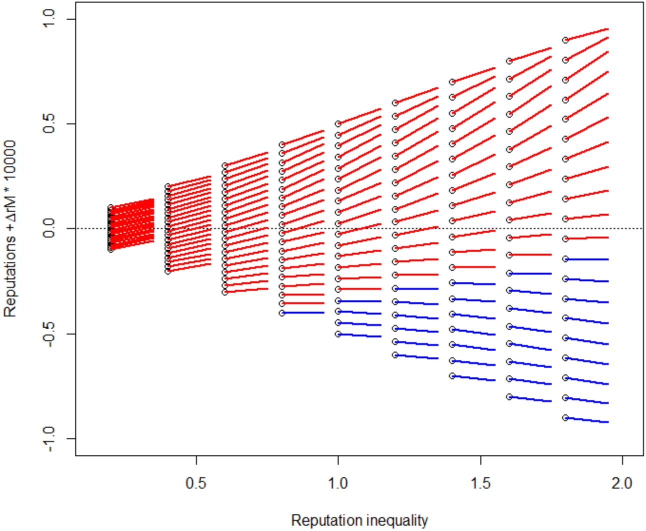


Lines: analytical approximation, dots: average over 100 million simulations

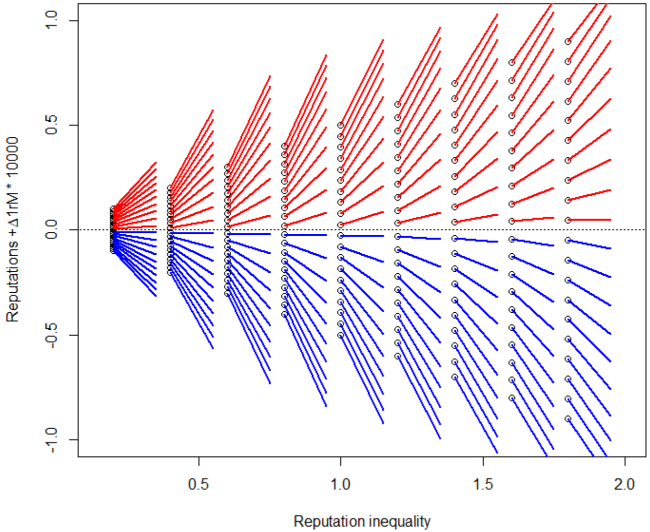
Studying the effect of inequalities on reputations with the average model

- $\Delta r_M(t)$, and the effect of its components for different levels of reputation inequalities for:
  - 20 agents without gossip
  - 20 agents with gossip
  - 40 agents without gossip
  - 40 agents with gossip

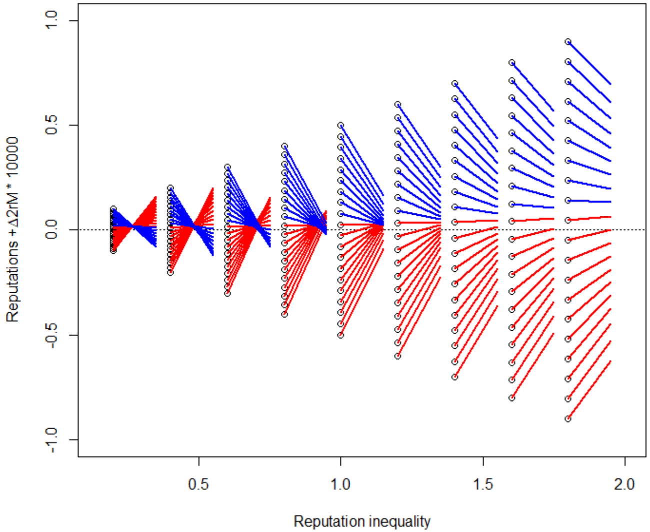
$\Delta r_M(t)$ for different reputation inequalities ($N_a = 20$)



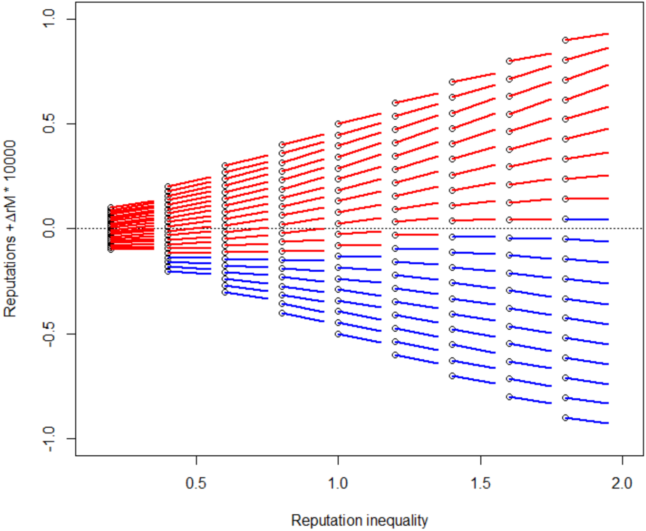
$\Delta_1 r_M(t)$ tends to increase inequalities ($N_a = 20$)



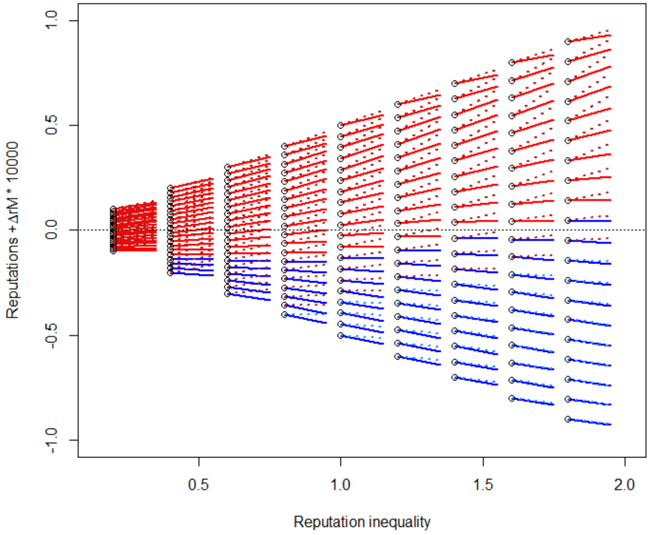
$\Delta_2 r_M(t)$ tends to decrease inequalities ($N_a = 20$)



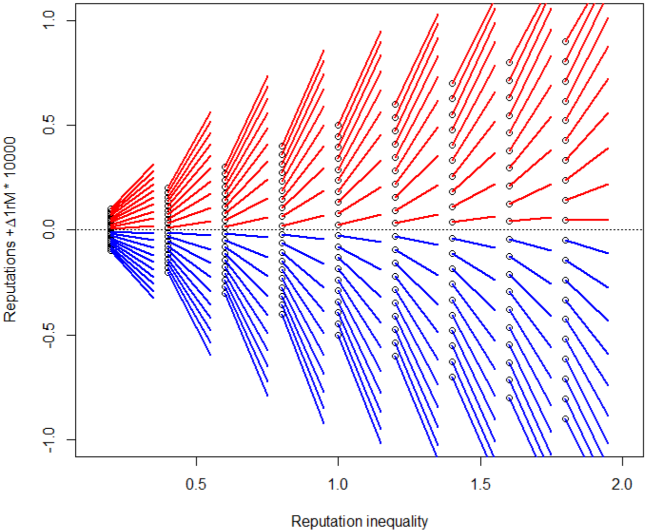
$\Delta r_M(t)$ with gossip ($N_a = 20$)



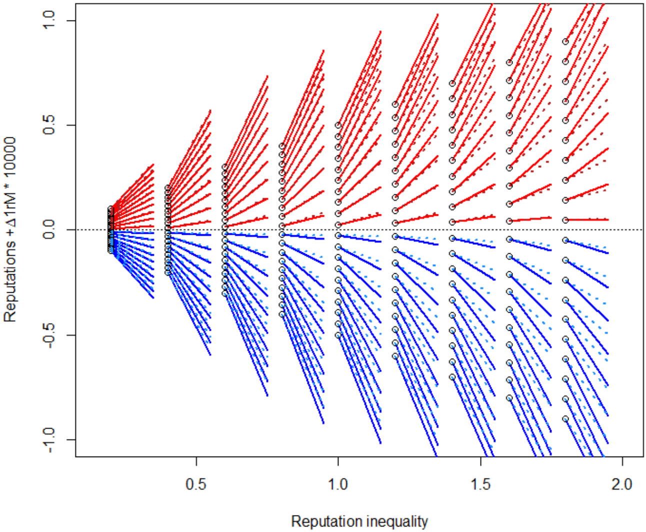
$\Delta r_M(t)$ with and without (dotted) gossip ($N_a = 20$)



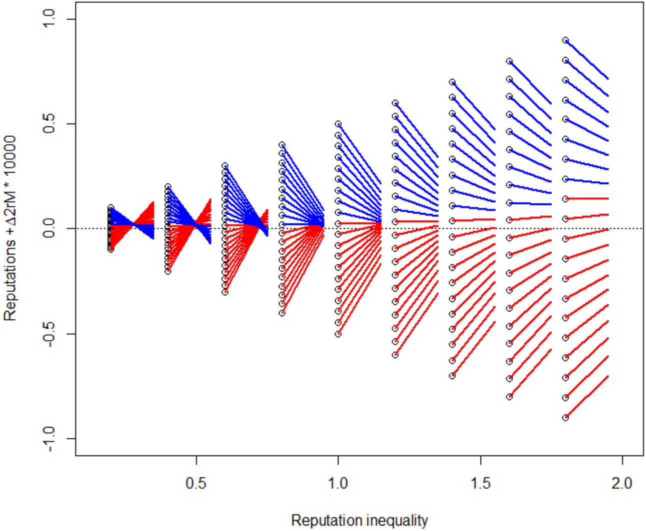
$\Delta_1 r_M(t)$ tends to increase inequalities ($N_a = 20$)



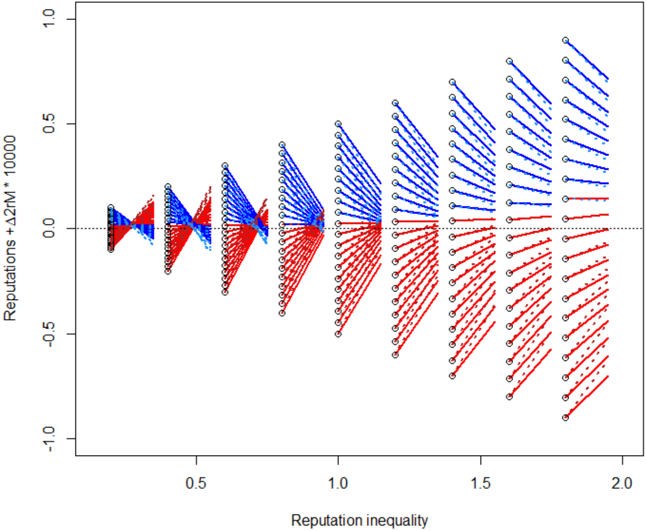
$\Delta_1 r_M(t)$ with and without gossip (dotted) ($N_a = 20$)



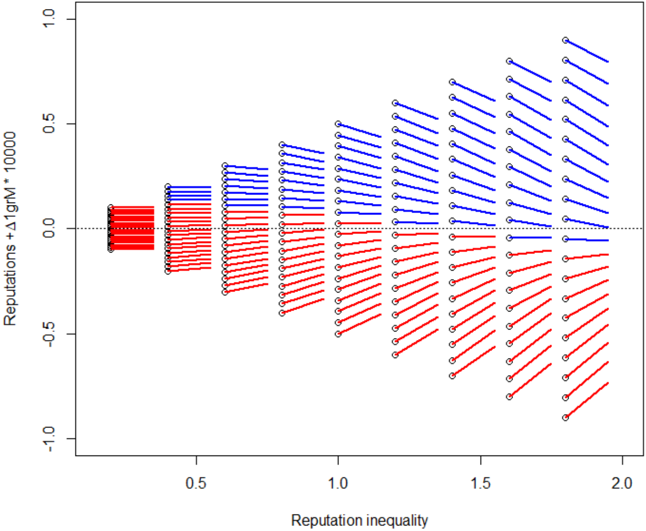
$\Delta_2 r_M(t)$ tends to decrease inequalities ($N_a = 20$)



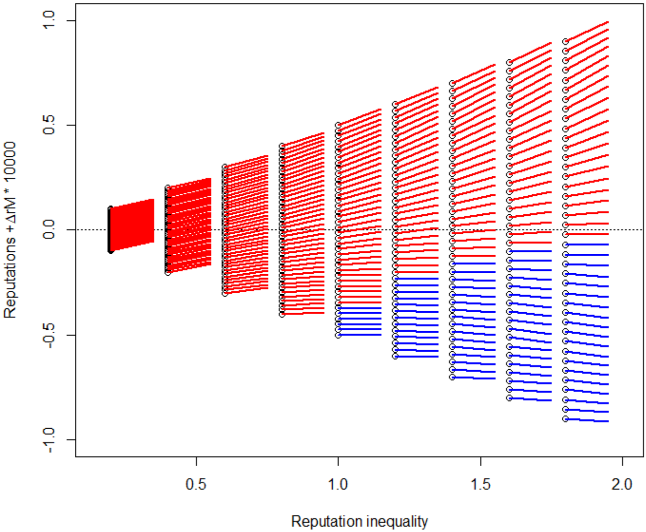
$\Delta_2 r_M(t)$ with and without (dotted) gossip ($N_a = 20$)



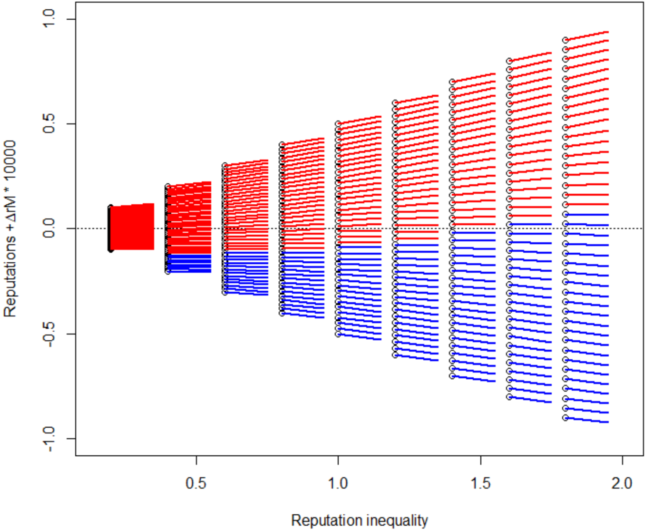
$\Delta_{1g} r_M(t)$ tends to decrease inequalities ($N_a = 20$)



$\Delta r_M(t)$ without gossip ($N_a = 40$)



$\Delta r_M(t)$ with gossip ($N_a = 40$)



Summary of results

- In the mode, beyond a threshold of perceived inequalities, these inequalities tend to increase: low reputations tend to decrease and high reputations tend to increase;
- When there is gossip, the threshold of inequalities is smaller if the inequalities are even bigger, a majority of reputations tends to decrease;
- Reducing perceived inequalities below a threshold increases average self-opinion and opinions about others.

Limitations / Perspectives

- Limitations:
  - Model limited to a small group, where everybody discusses with everybody
  - Other dynamics and heterogeneity between agents, not included in the model, could have much bigger effects
- Perspectives:
  - Extending the model to larger populations and introducing other processes (vanity, group identity).
  - Performing lab experiments checking predictions of the model (positive bias on self-opinions, negative bias on opinions about others)
  - Studying processes of gossip in social networks