

The Brexit Game

Uncertainty and Location Decision

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- 2 Methodology
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 - 4.1. Homogeneous Scenario
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Motivation

- This paper aims to analyze the strategic behavior of firms regarding location decision;
- **Brexit as a case study** to explain how the agglomerative effect may occur in an environment surrounded by uncertainty;
- Many fields in economics discuss firms' location strategy:
 - **Industrial Organization (IO)**
 - **Regional Economics**
 - **International Economics**

Motivation

- Few studies about spatial competition lying on the interface of Evolutionary Game Theory (EGT), Agent-Based Models (ABM) and Regional Science;
- We propose a model based on an input-output structure within which an evolutionary game is played;
- By doing so, the European Union was separated into two strategic regions: United Kingdom and rest of European Union.

Motivation

- Firms are competing in two different sectors:
 - (a) crop and animal production, hunting and related service activities;
 - (b) financial service activities, except insurance and pension funding.
- Payoff structures decide location decisions;
- If their goal is to **maximize market share**, they may assign greater weight to the variables (i) **market potential** and (ii) **productive integration**;
- If the goal is to **minimize costs**, firms may attribute greater weight to (iii) **labor costs** and (iv) **displacement cost**;

Motivation

- To increase realism, we developed a stochastic learning rule through a spatial Agent-based simulation (ABS) model;
- **Evolutionary Stable Strategy (ESS)**: economic situations involving dynamically interacting decisions in which firms can learn with their own choices over time.

Motivation

- EGT and ABS models may overcome some draw-backs of the existing literature about firms' location choice;
- Especially in situations in which the economic environment becomes a complex system - such as the **Brexit** event;
- **Bounded rationality assumption:** firms' strategic behavior goes beyond the payoff utility maximization;

Motivation

- Their decision-making is also **rule-guided**, relying essentially on institutional framework and organizational skills;
- There are many evidences that **routine behavior** of firms, together with their **learning abilities**, are related to agglomeration and regional growth differences;
- **Firms can learn** from their own mistakes through the trial-and-error dynamics and by mimicking their rivals' strategies.

Motivation

- **Competition** acts as a “**natural selection**” device: “intelligent” firms – the ones able to adapt their routine to the environment in which they compete – will spread in the long term;
- Then, firms’ persistent growth overtime is auto-correlated with their ability to adapt to the economic system;

Motivation

- In order to measure the consequences of Brexit for the economic activity, we consider the projection made from the Regional Economic analysis;
- The Input-Output analysis is the basis of our **Evolutionary Game**;
- **Evolutionary Stable Strategy (ESS)**: represents the optimal strategy location decision in the long term (players have no incentive to abandon);
- Following this framework, we simulate a game in a spatial structure to capture some of the behavioral aspects of firms' decision-making, as well as the clustering dynamics according to Brexit.

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2.1. Strategic Elements Evaluated in the Location Decision

- In order to develop the methodology, a game involving two types of firms is considered:
 - Type A* firm is situated in the rest of the European Union countries (EU) and assesses the possibility of migrating to the United Kingdom (UK);
 - Type B* firm is situated in (UK) and assesses the possibility of migrating to (EU);

2.1. Strategic Elements Evaluated in the Location Decision

- The payoffs are based on the (global) regional input-output matrix, **made available by the WIOD for 2014, with sector opening of 43 countries and 56 sectors;**
- Countries were aggregated so that the interregional input-output matrix **contains the same 56 sectors for three regions:** United Kingdom (GBR), Rest of Europe Union (RoEU) and Rest of the World (RoW).

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2.2. Input-Output Model

- The interregional input-output model can be represent from:

$$B^D X + F^D = X \quad (1)$$

$$B^M X + F^M = M \quad (2)$$

$$\mu B^D + \mu B^M + B_v = \mu \quad (3)$$

- Equations (1) and (2) represents the equilibrium conditions for the domestic and imported goods production, respectively. Equation (3) is the equilibrium condition that adds a constraint in the input-output coefficients.

2.2.1. Market Potential

- The market potential for non-Brexit cases will be expressed as the sum of the technical production coefficients for all i sectors of j countries of the European Union, in equation (3);
- Considering Brexit, the market potential is measured by the hypothetical extraction of the UK from the EU;
- UK's market potential is given by the sum of its own technical production coefficients;
- The market potential of the Rest of the EU countries correspond to the sum of the technical coefficients of all i sectors of j countries of the EU.

2.2.2. Productive Integration

- To measure the degree of productive integration we use the value-added indicator on gross exports (VAX ratios) as follows:

$$VAX = A'_v(I - A)^{-1}F^{DM} \quad (4)$$

- The higher the coefficient, the lower the degree of productive integration of the country in this sector.

2.2.3. Labor Cost

- The cost of the labor force should reflect the costs absorbed by firms taking into account a skilled Ψ_1 and unskilled Ψ_2 labor force for the same amount of production X in each sector i and in each country j ;
- The cost of the high and low skilled labor can be obtained by the product between the participation of each type of labor in the total remunerations and the gross value of the production of each sector i and for each country j , X_{ij} .

2.2.4. Displacement Costs

- Displacement costs are associated with differences in the share of high skilled labor and low skilled labor between different countries for the same quantity produced:

$$\mu = (\psi_{1ij} - \psi_{2ij})X_{ij}, \forall s \quad (5)$$

$$\mathbf{M} = (\Psi_{1ij} - \Psi_{2ij})X_{ij}, \forall (1 - s) \quad (6)$$

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3.1. Evolutionary Brexit Game Model

- In Evolutionary Games, departing from an initial condition, a unique stable steady state will be reached;
- We know how firms choose between multiple locational strategies;
- Then, we can study the robustness of firms' strategic behavior.

3.1. Evolutionary Brexit Game Model

- We assume: (a) bounded rationality; (b) a large population, n , of firms ($n \rightarrow \infty$) and (c) that firms can learn;
- Every period, a firm is randomly matched with another firm and they play a two-player game;
- Each firm is randomly assigned a locational strategy (EU or UK) at the initial time step ($t = 0$), which can be updated over time via the systematic interaction with other firms. Thus, one firm can mimic other firms' locational strategies.

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3.2. The Payoff Matrix

- Firms evaluate the possibility of relocation according to whether Brexit is likely to happen (s) or not ($1 - s$);
- Let the row player be one representative type A firm and the column player be one representative type B firm. The payoff matrix of the stage game is given by (7):

$$\begin{array}{cc}
 & \begin{array}{cc} UK & EU \end{array} \\
 \begin{array}{c} UK \\ EU \end{array} & \begin{pmatrix} \pi_{A,1} ; \pi_{B,1} & \pi'_{A,1} ; \pi'_{B,2} \\ \pi'_{A,2} ; \pi'_{B,1} & \pi_{A,2} ; \pi_{B,2} \end{pmatrix}
 \end{array} \quad (7)$$

3.2. The Payoff Matrix

- All the strategic variables discussed in the previous section are considered in the payoff matrix (7), which can be expressed as follows:

$$\pi_{A,1} = s[1/2 w_{\rho}\rho_{A,1} + 1/2 w_{\theta}\theta_{A,1} + 1/2 w_{\psi}(\psi_{1,A1} + \psi_{2,A1}) + w_{\mu}\mu_{A,1}] + (1-s)[1/2 W_{\mathbf{P}}\mathbf{P}_{A,1} + 1/2 W_{\Theta}\Theta_{A,1} + 1/2 W_{\Psi}(\Psi_{1,A1} + \Psi_{2,A1}) + W_{\mathbf{M}}\mathbf{M}_{A,1}]$$

$$\pi_{B,1} = s[1/2 w_{\rho}\rho_{A,1} + 1/2 w_{\theta}\theta_{A,1} + 1/2 w_{\psi}(\psi_{1,A1} + \psi_{2,A1})] + (1-s)[1/2 W_{\mathbf{P}}\mathbf{P}_{A,1} + 1/2 W_{\Theta}\Theta_{A,1} + 1/2 W_{\Psi}(\Psi_{1,A1} + \Psi_{2,A1})]$$

3.2. The Payoff Matrix

- Assume that $\eta_A = (\pi_{eu,uk}^A - \pi_{uk,uk}^A)$, $\eta_B = (\pi_{eu,uk}^B - \pi_{uk,uk}^B)$, $v_A = (\pi_{eu,uk}^A - \pi_{uk,eu}^A)$, and $v_B = (\pi_{eu,eu}^B - \pi_{uk,eu}^B)$.
- The matrix (8) is the basis of the evolutionary dynamics of the game:

$$\begin{array}{cc}
 & \begin{array}{cc} UK & EU \end{array} \\
 \begin{array}{c} UK \\ EU \end{array} & \left(\begin{array}{cc} \eta_A; \eta_B & 0; 0 \\ 0; 0 & v_A; v_B \end{array} \right)
 \end{array} \tag{8}$$

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3.3. The Replicator Dynamics (RD)

- The RD system for population A and B is given by:

$$\dot{f}_A = f_A(1 - f_A)[\eta_A f_B - v_A(1 - f_B)] \quad (9)$$

$$\dot{f}_B = f_B(1 - f_B)[\eta_B f_A - v_B(1 - f_A)] \quad (10)$$

- \dot{f}_A and \dot{f}_B represent the growth rate of the proportion of firms that adopt the first pure strategy UK within each population;
- $\dot{f}_A = \dot{f}_B = 0$ is a necessary condition for the stationarity of (9) and (10);
- For the stationary point to be asymptotically stable, the eigenvalues $\lambda_{1,2}$ of the matrix (Ω) evaluated at points that hold the condition $\dot{f}_A = 0$ and $\dot{f}_B = 0$ must have negative real parts.

Outline

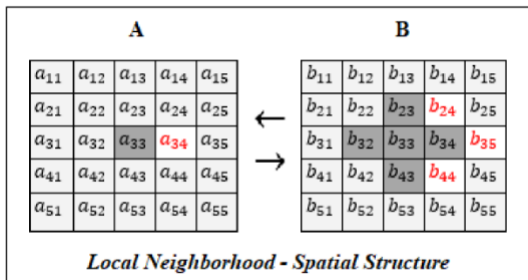
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3.4. Agent-Based Simulation Algorithm

- We now complement the EGT framework with a stochastic learning rule guided by the following equation:

$$w = \frac{V_j - V_i}{\text{max.payoff} - \text{min.payoff}} \quad (11)$$

- Thus, we apply this procedure in regular lattices, which directly impacts the value of w .



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4.1 Homogeneous Scenario

Homogeneous Scenario		Scenario II		Scenario III	
(s = 0.5)	(1-s = 0.5)	(s = 0.7)	(1-s = 0.3)	(s = 0.7)	(1-s = 0.3)
$w_\rho = 0.25$	$W_{\mathbf{P}} = 0.25$	$w_\rho = 0.45$	$W_{\mathbf{P}} = 0.45$	$w_\rho = 0.05$	$W_{\mathbf{P}} = 0.05$
$w_\theta = 0.25$	$W_{\mathbf{\Theta}} = 0.25$	$w_\theta = 0.45$	$W_{\mathbf{\Theta}} = 0.45$	$w_\theta = 0.05$	$W_{\mathbf{\Theta}} = 0.05$
$w_\psi = 0.25$	$W_{\mathbf{\Psi}} = 0.25$	$w_\psi = 0.05$	$W_{\mathbf{\Psi}} = 0.05$	$w_\psi = 0.45$	$W_{\mathbf{\Psi}} = 0.45$
$w_\mu = 0.25$	$W_{\mathbf{M}} = 0.25$	$w_\mu = 0.05$	$W_{\mathbf{M}} = 0.05$	$w_\mu = 0.45$	$W_{\mathbf{M}} = 0.45$

Table 1. Summary of the weighted factors considered in each Scenario.

- To generate the results, we assign weights to the variables obtained from the input-output analysis for sectors 1 and 41.

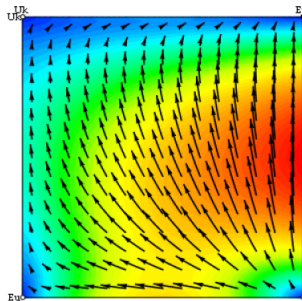
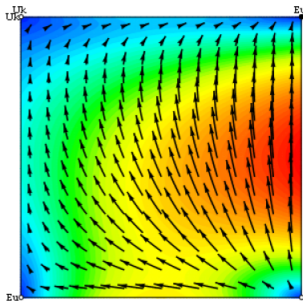
4.1 Homogeneous Scenario

Sector 1

$$\begin{array}{l}
 UK \\
 EU
 \end{array}
 \begin{pmatrix}
 UK & EU \\
 0.14; -0.1 & \mathbf{0;0} \\
 0;0 & -0.62; -0.38
 \end{pmatrix}$$

Sector 41

$$\begin{array}{l}
 UK \\
 EU
 \end{array}
 \begin{pmatrix}
 UK & EU \\
 0.16; -0.07 & \mathbf{0;0} \\
 0;0 & -0.64; -0.41
 \end{pmatrix}$$



4.1 Homogeneous Scenario

- The homogeneous scenario features a unique balance for both Sector 1 and Sector 41: $\Phi^{ESS} = (UK, EU)$;
- We observed firms that were initially operating in the *EU* updating its locational decision to *UK*.
- On the other hand, firms established in *UK*, observing this movement, will do the opposite movement, that is, they will start to migrate to *EU*;
- The economic intuition of this equilibrium is that firms has some benefits when incurring at the displacement cost, since it relocates in an unsaturated market.

4.1 Homogeneous Scenario

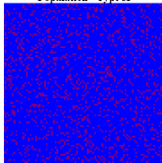
- By considering a low level of spatial division, in addition to not providing greater relevance to any of the factors considered in the analysis, there is a need to better evaluate the trade-off between the advantages of agglomeration (ρ and θ) and production costs (ψ and μ);
- In addition, another relevant issue is to know more precisely in which of the EU countries firms will decide to locate;
- Thus, we apply the ABS model to better evaluate the dynamic equilibrium of the spatial competition among firms.

4.1 Homogeneous Scenario

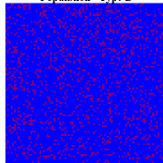
Sector 1

$$\begin{array}{l}
 UK \\
 EU
 \end{array}
 \begin{pmatrix}
 UK & EU \\
 0.14; -0.1 & 0; 0 \\
 0; 0 & -0.62; -0.38
 \end{pmatrix}$$

Population - Type A



Population - Type B

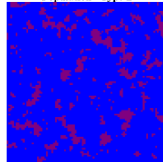


MCS 1

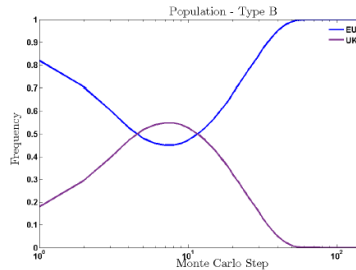
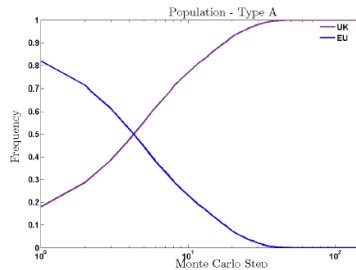
Population - Type A



Population - Type B



MCS 30



Outline

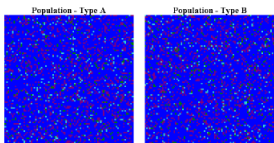
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4.2 Scenario II

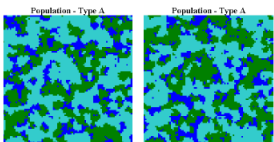
Emphasis on (ρ, \mathbf{P}) and (θ, Θ) - Sector 1

Sector 1

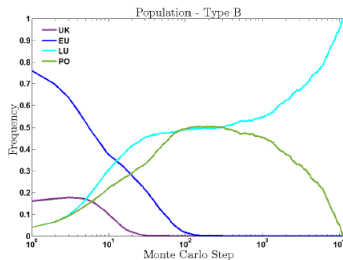
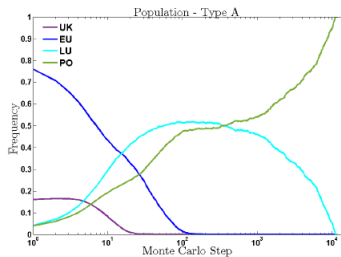
	UK	EU	LU	PO
UK	0.29; 0.32	0.60; 0.77	0.60; 0.90	0.60; 0.84
EU	0.76; 0.63	0.38; 0.39	0.76; 0.90	0.76; 0.84
LU	0.90; 0.63	0.90; 0.77	0.45; 0.45	0.90; 0.84
PO	0.84; 0.63	0.84; 0.77	0.84; 0.90	0.43; 0.43



MCS I



MCS 40

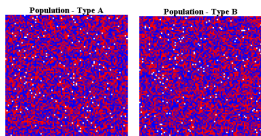


4.2 Scenario II

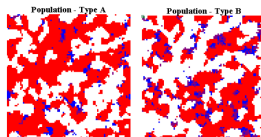
Emphasis on (ρ, \mathbf{P}) and (θ, Θ) - Sector 41

Sector 41

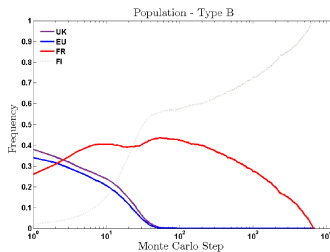
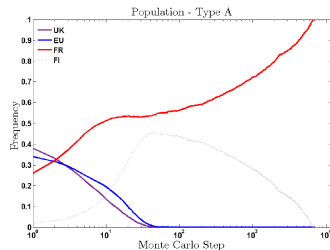
	UK	EU	FR	FI
UK	0.32; 0.35	0.64; 0.67	0.64; 0.84	0.64; 0.90
EU	0.67; 0.67	0.33; 0.33	0.67; 0.84	0.67; 0.90
FR	0.88; 0.67	0.84; 0.67	0.43; 0.43	0.84; 0.90
FI	0.90; 0.67	0.90; 0.67	0.90; 0.84	0.45; 0.45



MCS I



MCS 40



Outline

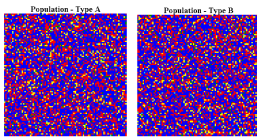
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4.3 Scenario III

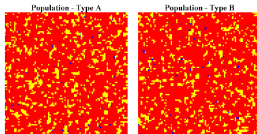
Emphasis on (ψ, μ) and (Ψ, \mathbf{M}) - Sector 1

Sector 1

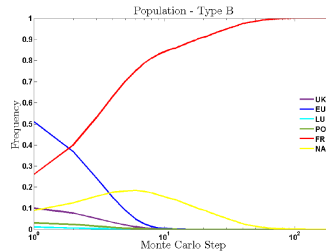
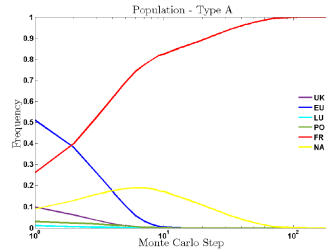
	UK	EU	LU	PO	FR	NL
UK	-0.08; 0.21	-0.05; 0.20	-0.05; 0.10	-0.05; 0.20	-0.05; 0.97	-0.05; 0.86
EU	0.13; 0.24	0.09; 0.16	0.13; 0.10	0.13; 0.20	0.13; 0.97	0.13; 0.86
LU	0.10; 0.24	0.10; 0.20	0.05; 0.05	0.10; 0.20	0.10; 0.97	0.10; 0.86
PO	0.20; 0.24	0.20; 0.20	0.20; 0.10	0.16; 0.16	0.20; 0.97	0.20; 0.86
FR	0.97; 0.24	0.97; 0.20	0.97; 0.10	0.97; 0.20	0.93; 0.93	0.97; 0.86
NL	0.86; 0.24	0.86; 0.20	0.86; 0.10	0.86; 0.20	0.86; 0.97	0.83; 0.83



MCS 1



MCS 10

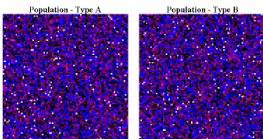


4.3 Scenario III

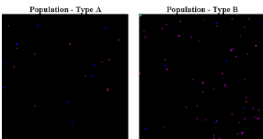
Emphasis on (ψ, μ) and (Ψ, M) - Sector 41

Sector 41

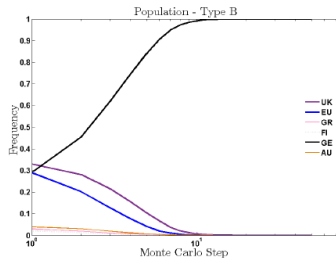
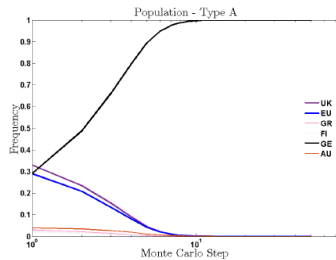
	UK	EU	GR	FI	GE	AU
UK	0.04; 0.30	0.08; 0.13	0.08; 0.08	0.08; 0.08	0.08; 1.10	0.08; 0.22
EU	0.10; 0.33	0.06; 0.09	0.10; 0.08	0.10; 0.08	0.10; 1.10	0.08; 0.22
GR	0.09; 0.33	0.08; 0.13	0.04; 0.04	0.08; 0.09	0.08; 1.10	0.08; 0.22
FI	0.09; 0.33	0.09; 0.13	0.09; 0.08	0.04; 0.04	0.09; 1.10	0.09; 0.22
GE	1.10; 0.33	1.10; 0.13	1.10; 0.08	1.10; 0.09	1.06; 1.06	1.10; 0.22
AU	0.22; 0.33	0.22; 0.13	0.22; 0.08	0.22; 0.09	0.22; 1.10	0.18; 0.18



MCS I



MCS 10



5. Conclusion

- Brexit may modify the distribution of the relevant geographic markets on the European continent;
- If firms are giving more emphasis to the agglomerative advantages, associated displacement costs are irrelevant. Firms see greater benefits when moving from the host country;
- When firms are giving more emphasis to cost minimization, they counterpoint these factors (indirectly) with the relevance of productive integration and market potential in their decision-making process. Then, firms can see greater benefits in splitting the same market.
- In sectors highly dependent on natural resources and traditionally benefited by fiscal subsidies, Brexit might lead firms to search for unsaturated markets;

Outline

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 - 3.1. Evolutionary Brexit Game Model
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5.1. Future Research

- There are many paths to broaden this research:
 - (a)** The generality and flexibility of our agent-based model would allow, for instance, to incorporate Russia as one of the strategic location to be chosen by the firms;
 - (b)** Thus, it would be possible to discuss international trade policies – such as trade barriers and free trade areas – which might provide insights into how these measures would impact firms' locational decision in these regions;
 - (c)** We could focus the discussion on firms locational decision by considering only the strategic interaction between Russia and its border with the EU member states: Estonia, Finland, Latvia, Lithuania, and Poland.